

# **Script**

## **4.254 Linear Models and Variance Analysis**

**Second version, work in progress**

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