Discussion of "VIX Dynamics with Stochastic Volatility of Volatility" by Andreas Kaeck and Carol Alexander

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In a nutshell

- Describe physical dynamics of VIX in reduced form. Try
  - Affine and non-affine one-factor models
  - Affine two-factor model
  - With and without jumps
- Perform specification analysis
- Discuss implications on risk management and VIX derivatives

Contribution:

- Use more data (20 years)
- Add stochastic volatility
- Add non-affine models $\leftrightarrow$ MCMC framework
- Use posterior predictive p-values for specification analysis

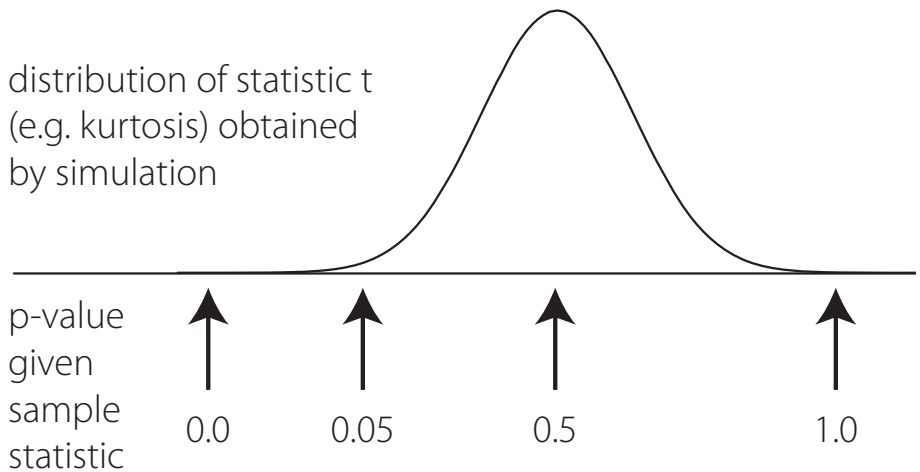
Related literature:

- Kaeck, Alexander (wp 2010). Stochastic volatility jump-diffusions for equity index dynamics.
- Psychoyios, Dotsis, Markellos (Rev Quant Fin Acc 2009). A jump diffusion model for VIX volatility options and futures.

Methods

Predictive p-values

- **Draw** parameter vector from posterior distribution
  (taking parameter uncertainty into account; independent draws)
- **Simulate**
- **Calculate** interesting statistics
  (moments, dispersion, extrema; almost all non-robust)
- **Compare** distribution of these statistics to observed value
  Good: $p \approx 0.5$ (i.e. model can reproduce observed statistics)



Results

Specification analysis

- Method of predictive p-values delivers valuable insights
- Modeling VIX returns (log model) beats VIX levels
- Non-affine models beat affine ones
- Jumps beat pure diffusive models
- Best: stochastic vol-of-VIX model

Scenario analysis

- Crisis of 2008 within 99.9% percentile? Yes!

Derivatives

- Can we reproduce **average** term structure of VIX futures? Yes!
- But: **variation** in term structures and option prices cannot be reproduced

Praise

Paper

- A joy to read

Specification analysis

- Clear ranking of models
- Application of predictive p-values goes beyond pure model selection and provides valuable insights in the functioning and limits of each model

Scenario analysis

- Crisis scenario is beautiful illustration of different **feasible regions** of the models

Comments

Option pricing example

- Zero market price of risk debatable
- Level models: option prices driven by mean reversion (evidence?)
  Low VIX $\rightarrow$ high option price; high VIX $\rightarrow$ low option price
- OTM options capture mostly (bias in) kurtosis

Model	Call price	p-value of kurt
Level, b=0.5, expJ	0.00	0.997
Log, b=0, normJ	0.08	0.98
Level, b=2, expJ	0.03	0.90
Log, SVV, noJ	0.44	0.83
Log, SVV, normJ	0.43	0.79
Log, b=1, expJ	0.55	0.52

- Represent prices as Black-Scholes implied volatilities?
- Show achievable ranges of prices/implied volatilities?

Simulation exercise (1)

Benefit of SV model is very indirect

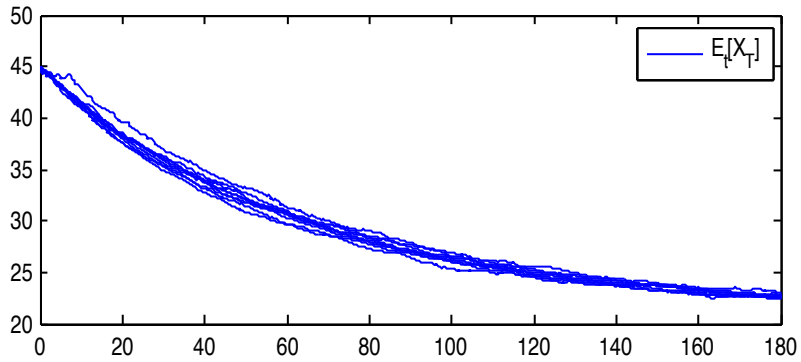
- Stochastic volatility allows to reduce jump intensity (possibly through leverage effect)
- But: ultra-fast mean reversion of V_t (10 days; $\kappa_v = 0.097$)
  $\rightarrow$ **cannot** reproduce different term structures of VIX future
  $\rightarrow$ **could** reproduce different short-term option prices

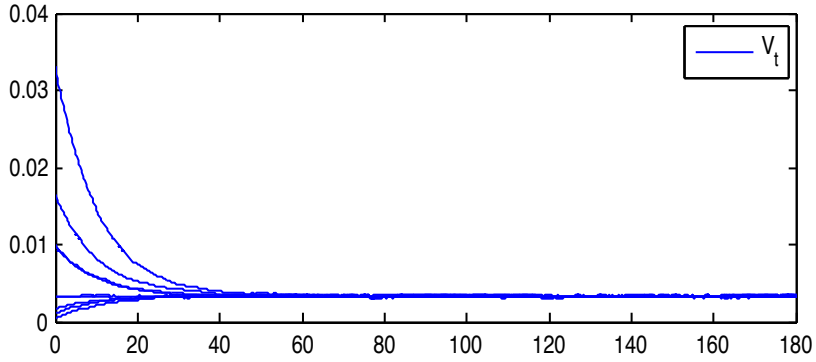
Simulating the model

- SV-J model
- Point-estimates from paper
- $X_0 = 45\%$, vary V_0
- $V_0/\theta_v = \{0.5, 1.0, 3.0, 5.0\}$ ($\approx$ observed range)

Simulation exercise (2)

V_0 **cannot** change VIX term structure





V_0 **can** change short-term option prices

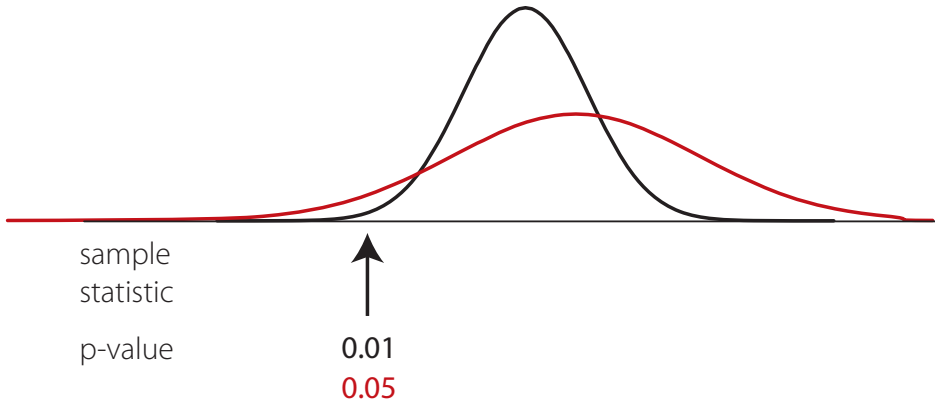
- Example: european call $\tau = 30d$ and $\tau = 180d$; $K/X_0 = 1.5$; $\mathbb{P} = \mathbb{Q}$

V_0/θ_v	0.5	1.0	3.0	5.0	(paper)
$C(\cdot, V_0)_{\tau=30d}$	0.2217	0.3244	0.7847	1.4706	0.43
$C(\cdot, V_0)_{\tau=180d}$	0.1609	0.1321	0.1525	0.1332	

- VIX-process is mean reverting $\rightarrow$ **long option cheaper than short**
- Most other option pricing problems: underlying is a martingale

Points that may need clarification:

Does the predictive p-value favor wide posterior distributions?



- Which distribution should we prefer?
- In the limit, an **uninformed** posterior distribution would have the "perfect" p-value of 0.5 **regardless of the parameter value**

Special role of the VIX as volatility gauge

- Is there such a thing as a "VIX process"?
- Joint analysis of SP500 and VIX?
  - SP500 options contain volatility risk premium
  - VIX options contain a vol-of-vol risk premium
  - How are they related?